

# The Embedded Muse 39

Editor: Jack Ganssle ([jack@ganssle.com](mailto:jack@ganssle.com))

October 21, 1999

## Floating Point Approximations

I was surprised to find that one of my favorite programming books is no longer in print. "Computer Approximations" by J.F. Hart (John Wiley & Sons, 1968, ISBN 0-88275-642-7) is the bible of floating point approximations. Amazon.com claims they might be able to find used copies, and any decent university library will have a copy.

C libraries include all of the standard math functions for trig, exponentiation, and the like. That's not much help to assembly-language programmers or C coders who had to delete the math library to save space. And, the C libraries are typically aimed at the mass of developers, offering high precision answers despite long execution times. Need a particularly fast trig function, and are you willing to sacrifice some precision? Hart's book is the place to find an appropriate algorithm.

Hart's book gives polynomial solutions for all sorts of functions, including logs, trig, roots, etc. He also presents a number of variants, so you can select for a longer polynomial of high accuracy (and slower execution) or a shorter one that solves quickly but not so accurately. For instance, the COSINE function can be calculated to about 5 decimal digits of accuracy by:

$$\text{Cos}(x) = .9999932946 - .4999990534 * x^{**2} + .0414877472 * x^{**4} - .00127120948 * x^{**6}$$

Need more speed? Try the following which gives 3.2 digits of accuracy:

$$\text{Cos}(x) = .99940307 - .49558072 * x^{**2} + .03679168 * x^{**4}$$

These approximations are valid for the range of 0 to 90 degrees. The argument "x" is in radians.

He gives 46 different approximations for the cosine alone, with accuracies ranging from 2 to 23 decimal digits. For low resolution integer-only applications scale his coefficients to integers and save the space of lookup tables.

Consider square roots: most of us write these as iterative algorithms that eat up tons of execution time. All of Hart's square root algorithms (88 variants are presented) use polynomial solutions, that execute in more or less fixed times.

*Copyright 2000 by The Ganssle Group. All Rights Reserved. You may distribute this for non-commercial purposes. Contact us at [info@ganssle.com](mailto:info@ganssle.com) for more information.*

I've found references in runtime libraries to Hart's book going all the way back to the PDP-11 Fortran products from DEC 30 years ago. It's the best source of algorithms for these sorts of problems.

Hart will frustrate some readers as he presents derivations that are deeply mathematical. I find the tables of polynomial coefficients tremendously useful though, and rarely bother with the math details.

## **Thought for the Week**

Thad Badowski sent this gem along. I thought it was particularly useful considering the recent loss of the Mars Climate Observer spacecraft due to a metric conversion problem.

"Useful Metric Conversions"

Americans (defined as residents of the USA) frequently have problems with metric conversions. In an attempt to clarify the conversion process I now submit some "Useful Metric Conversions."

1 million microphones = 1 megaphone  
2000 mockingbirds = two kilomockingbirds  
10 cards = 1 decacards  
1 millionth of a fish = 1 microfiche  
453.6 graham crackers = 1 pound cake  
1 trillion pins = 1 terrapin  
10 rations = 1 decoration  
100 rations = 1 C-ration  
10 millipedes = 1 centipede  
3 1/3 tridents = 1 decadent  
2 monograms = 1 diagram  
8 nickels = 2 paradigms  
2 wharves = 1 paradox

## **About The Embedded Muse**

The Embedded Muse is an occasional newsletter sent via email by Jack Ganssle. Send complaints, comments, and contributions to him at [jack@ganssle.com](mailto:jack@ganssle.com).

To subscribe, send a message to [majordomo@ganssle.com](mailto:majordomo@ganssle.com), with the

*Copyright 2000 by The Ganssle Group. All Rights Reserved. You may distribute this for non-commercial purposes. Contact us at [info@ganssle.com](mailto:info@ganssle.com) for more information.*

**The Ganssle Group, [www.ganssle.com](http://www.ganssle.com)**

words “subscribe embedded *your-email-address*” in the body. To unsubscribe, change the message to “unsubscribe embedded *your-email-address*”.

The Embedded Muse is supported by The Ganssle Group, whose mission is to help embedded folks get better products to market faster. We offer seminars at your site offering hard-hitting ideas - and action - you can take now to ***improve firmware quality and decrease development time***. Contact us at [info@ganssle.com](mailto:info@ganssle.com) for more information.

*Copyright 2000 by The Ganssle Group. All Rights Reserved. You may distribute this for non-commercial purposes. Contact us at [info@ganssle.com](mailto:info@ganssle.com) for more information.*

**The Ganssle Group, [www.ganssle.com](http://www.ganssle.com)**